MENIIT
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JEE MAINS-2012

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3** hours duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use **Blue/Black Ball Point Pen only** for writing particulars/marking responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **21 – 23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**
12. The CODE for this Booklet is **C**. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

PART-A-CHEMISTRY

1. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, Ka of this acid is:

(A) 1×10^{-5} (B) 1×10^{-7} (C) 3×10^{-1} (D) 1×10^{-3}

Ans. [A]

Sol. $pH = 3 \Rightarrow [H^+] = 10^{-3}\ M = \alpha\ C$

$$\alpha = \frac{10^{-3}}{C} = \frac{10^{-3}}{0.1} = 0.01 << 1$$

$\therefore Ka = \alpha^2 C = (0.01)^2 \times 0.1 = 1 \times 10^{-5}$

2. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide?

(A) $[Cr\ (en)\ Br_4]^-$ (B) $[Cr\ (en)\ Br_2]^-$ (C) $[Cr\ (en)_3]\ Br_3$ (D) $[Cr\ (en)_2\ Br_2]\ Br$

Ans. [D]

Sol. dibromodobis(ethylene diamine) chromium (III) bromide

2Br ↓ en ↓ Cr^{+3} ↓ Br^-

$[Cr(en)_2Br_2]Br$

3. Which method of purification is represented by the following equation :

$$Ti(s) + 2I_2(g) \xrightarrow{523K} TiI_4\ (g) \xrightarrow{1700K} Ti(s) + 2I_2(g)$$

(A) Poling (B) Van Arkel (C) Zone refining (D) Cupellation

Ans. [B]

Sol. Van Arkel Method

4. The compressibility factor for a real gas at high pressure is

(A) 1 + pb/RT (B) 1 – pb/RT (C) 1 + RT/pb (D) 1

Ans. [A]

Sol. At high pressure : $\left(P + \frac{a}{V^2}\right) \approx P$

$$\therefore \left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Reduce to P(V –b) = RT

or PV = RT + bP

$$\therefore Z = \frac{PV}{RT} = 1 + \frac{bP}{RT}$$

5. The increasing order of the ionic radii of the given isoelectronic species is :

(A) $Ca^{2+}, K^+, Cl^-, S^{2-}$ (B) $K^+, S^{2-}, Ca^{2+}, Cl^-$ (C) $Cl^-, Ca^{2+}, K^+, S^{2-}$ (D) $S^{2-}, Cl^-, Ca^{2+}, K^+$

Ans. [A]

Sol. Ca^{2+}, K^{+}, Cl^{-}, S^{2-} isoelectronic series

6. The species which can best serve as an initiator for the cationic polymerization is :

(A) $AlCl_3$ (B) BuLi (C) LiAlH4 (D) HNO_3

Ans. [A]

Sol. For cationic polymerization the best reagent will be $AlCl_3$

7. The molecule having smallest bond angle is :

(A) $SbCl_3$ (B) PCl_3 (C) NCl_3 (D) $AsCl_3$

Ans. [A]

Sol. NCl_3 Same group bond angle

PCl_3 decrease on moving top

$AsCl_3$ ↓ to bottom.

$SbCl_3$

8. The equilibrium constant (K_C) for the reaction $N_2(g) + O_2(g) \rightarrow 2NO(g)$ at temperature T is 4×10^{-4}. The value of K_C for the reaction, $NO(g) \rightarrow 1/2N_2(g) + 1/2\ O_2(g)$ at the same temperature is :

(A) 4×10^{-4} (B) 50.0 (C) 0.02 (D) 2.5×10^{2}

Ans. [B]

Sol. $N_2(g) + O_2(g) \rightarrow 2NO(g)$; $k_1 = 4 \times 10^{-4}$

$NO(g) \rightarrow 1/2N_2(g) + 1/2\ O_2(g)$; k_2 =?

$$k_2 = \frac{1}{\sqrt{k_1}} = 50$$ **Ans.**

9. Iron exhibit + 2 and + 3 oxidation states. Which of the following statements about iron is incorrect?

(A) Ferrous compounds are less volatile than the corresponding ferric compounds.

(B) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

(C) Ferrous oxide is more basic in nature than the ferric oxide.

(D) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.

Ans. [B]

Sol. Based on Facts

10. The electrons identified by quantum numbers n and ℓ :

(a) n = 4, ℓ = 1 (b) n = 4, ℓ = 0 (c) n = 3, ℓ = 2 (d) n = 3 = ℓ = 1

can be placed in order of increasing energy as :

(A) (b) < (d) < (a) < (c) (B) (a) < (c) < (b) < (d) (C) (c) < (d) < (b) < (a) (D) (d) < (b) < (c) < (a)

Ans. [4]

Sol.

	Value of (n + ℓ)
(a) n = 4, ℓ = 1 4p	4 + 1 = 5
(b) n = 4, ℓ = 0 4s	4 + 0 = 4

(c) n = 3, ℓ = 2 3d 3 + 2 = 5

(d) n = 3, ℓ = 1 3p 3 + 1 = 4

orbital having more (n + ℓ) value has more energy if value of (n + ℓ) is same then orbital having lower value of n has less energy.

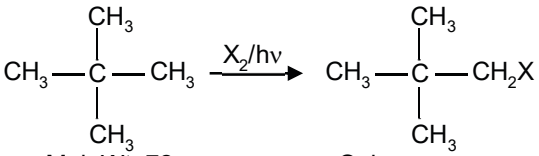
3p < 4s < 3d < 4p

11. Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alkyl halide?

(A) Isohexane (B) Neohexane

(C) Tertiary butyl chloride (D) Neopentane.

Ans. [D]

Sol. 

Mol. Wt=72
Neo pentane

Only one mono
Substituted alkyl halide

12. Which one of the following statement is correct?

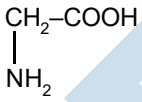
(A) All amino acids except glycine are optically active.

(B) All amino acids except glutamic acid are optically active.

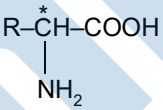
(C) All amino acids except lysine are optically active.

(D) All amino acids are optically active.

Ans. [A]

Sol. Glycine 

Optically inactive

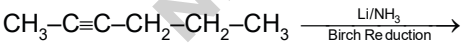
Except this 

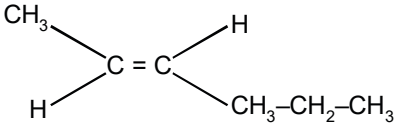
Optically inactive

13. 2–Hexyne gives trans-2-Hexene on treatment with :

(A) $Pd/BaSO_4$ (B) $LiAlH_4$ (C) Pt/H_2 (D) Li/NH_3

Ans. [D]

Sol. 



Trans-2-hexene

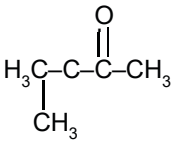
14. Iodoform can be prepared from all except :

(A) 3-Methyl-2-butanone (B) Isobutyl alcohol

(C) Ethyl methyl ketone (D) Isopropyl alcohol

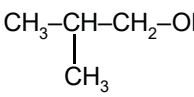
Ans. [B]

Sol. 3 - methyl - 2- butanone



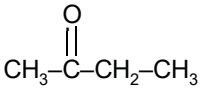
it will give iodoform

Isobutyl alcohol

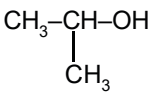


this will not give iodoform

Ethyl methyl ketone



It will give



iso propyl alcohol it will give iodoform

15. The incorrect expression among the following is :

(A) $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$ (B) $K = e^{-\Delta G^\circ/RT}$

(C) $\frac{\Delta G_{system}}{\Delta S_{total}} = -T$ (D) In isothermal process, $w_{reversible} = -nRT \ln \frac{V_f}{V_i}$

Ans. [A]

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -RT \ln K$

$\therefore \ln K = \frac{T\Delta S^\circ - \Delta H^\circ}{RT}$

16. The standard reduction potentials for Zn^{2+} / Zn, Ni^{2+} / Ni, and Fe^{2+} / Fe are –0.76, –0.23 and 0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when :

(A) X = Fe, Y = Zn (B) X = Zn, Y = Ni (C) X = Ni, Y = Fe (D) X = Ni, Y = Zn

Ans. [B]

Sol. $E^\circ_Y > E^\circ_X$

Hence, Y = Ni and X = Zn

17. Lithium forms body centered cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :

(A) 240 pm (B) 152 pm (C) 75 pm (D) 300 pm

Ans. [B]

Sol. For BCC : $\sqrt{3}a = 4r$

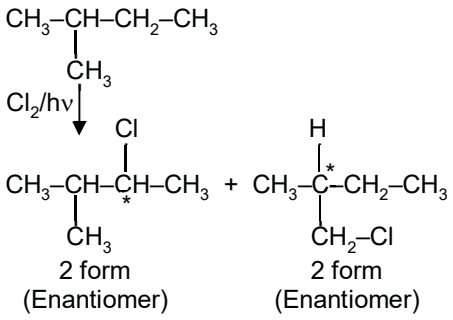
$\therefore\ r = \frac{\sqrt{3}}{4} \times 351 = 152pm$ **Ans.**

18. How many chiral compounds are possible on monochlorination of 2-methyl butane ?

(A) 4 (B) 6 (C) 8 (D) 2

Ans. [A]

Sol.



$\therefore$ Total 4 Chiral molecules

19. K_f for water is 1.86 K kg mol^{-1}. If you automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8°C?

(A) 39 g (B) 27 g (C) 72 g (D) 93 g

Ans. [4]

Sol. $\Delta T_f = K_f \times m$ or $2.8 = 1.86 \times \frac{W/62}{1}$

$\therefore$ W = 93

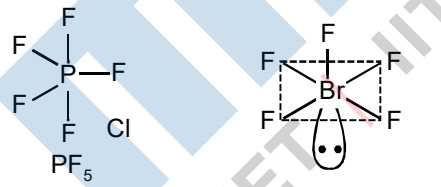
20. In which of the following pairs the two species are not isostructural?

(A) PF_5 and BrF_5 (B) AlF_6^{3-} and SF_6

(C) CO_3^{2-} and NO_3^- (D) PCl_4^+ and $SiCl_4$

Ans. [A]

Sol.



21. For a first order reaction, (A) $\rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M, is :

(A) 3.47×10^{-5} M/min (B) 1.73×10^{-4} M/min

(C) 1.73×10^{-5} M/min (D) 3.47×10^{-4} M/min

Ans. [D]

Sol. 0.1 M $\xrightarrow{t_{1/2}}$ 0.05 M $\xrightarrow{t_{1/2}}$ 0.025 M

$\therefore t_{1/2} = 40/2 = 20$ min

Now, $r = K[A] = \frac{0.693}{20\,min} \times 0.01$

$= 3.47 \times 10^{-4}$ M/ min **Ans.**

22. Ortho-Nitrophenol is less soluble in water than p- and m-Nitrophenols because :

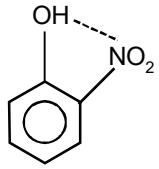
(A) o-Nitrophenol shows intermolecular H-bonding

(B) Melting point of o-Nitrophenol is lower than those of m-and p-isomers.

(C) o-Nitrophenol is more volatile in steam than those of m-and p-isomers.

(D) o-Nitrophenol shows intramolecular H-bonding

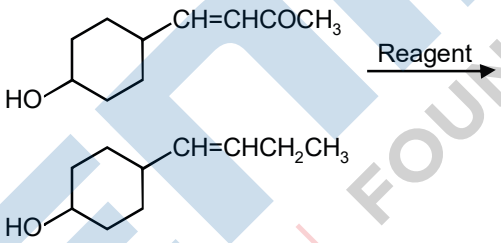
Ans. [D]

Sol. 

Due to intramolecular H-bonding its H-bonding with H_2O decreases.

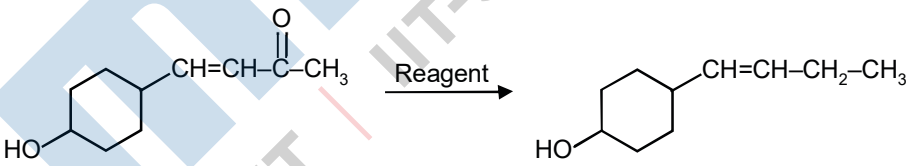
$\therefore$ Solubility became 69 less.

23. In the given transformation, which of the following is the most appropriate reagent ?



(A) Na, Liq. NH_3 (B) $NaBH_4$ (C) NH_2NH_2, $\overset{1}{O}H$ (D) Zn – Hg/HCl

Ans. [3]

Sol. 

Reagent should not effect – OH and C=C

$\therefore$ Alkaline medium is best suited

$\therefore$ Wolf - Khischner $NH_2NH_2/\overline{OH}$ is most appropriate.

24. According to Freundlich adsorption isotherm, which fo the following is correct?

(A) $\frac{x}{m} \propto p^{1/n}$

(B) $\frac{x}{m} \propto p^{0}$

(C) $\frac{x}{m} \propto p^1$

(D) All the above the correct for different ranges of pressure.

Ans. [A]

Sol. According to Freundlich, $\frac{x}{m} \times P^{1/n}$

Note : value of n is constant for a particular system from Freundlich isotherm.

25. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is :

(A) 1.02 M (B) 2.05 M (C) 0.50 M (D) 1.78 M

Ans. [B]

Sol. $\text{Molarity} = \frac{n_{solute}}{V_{solution}} = \frac{120/60}{(1000+120)/1.15} \times 1000 \approx 2.05\,M$ **Ans.**

26. Which of the following on thermal decomposition yields a basic as well as an acidic oxide ?

(A) $CaCO_3$ (B) NH_4NO_3 (C) $NaNO_3$ (D) $KClO_3$

Ans. [A]

Sol. $CaCO_3 \rightarrow \underset{\text{Basic}}{CaO} + \underset{\text{Acidic}}{CO_2}$

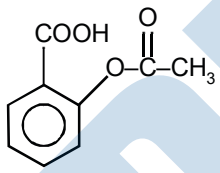
27. Aspirin is known as :

(A) Acetyl salicylate (B) Methyl salicylic acid

(C) Acetyl salicylic acid (D) Phenyl salicylate

Ans. [C]

Sol.



Aspirin

Acetyl Salicylic Acid

28. Which of the following compounds can be detected by Molisch's test?

(A) Amines (B) Primary alcohols (C) Nitro compounds (D) Sugars

Ans. [4]

Sol. Molisch Test is for Carbohydrates

$\therefore$ Sugar can be detected by Molisch Test.

29. What is DDT among the following :

(A) Biodegradable pollutant (B) Non-biodegradable pollutant

(C) Greenhouse gas (D) A fertilizer

Ans. [B]

30. Very pure hydrogen (99.9%) can be made by which of the following processes?

(A) Electrolysis of water

(B) Reaction of salt like hydrides with water

(C) Reaction of methane with steam

(D) Mixing natural hydrocarbons of high molecular weight

Ans. [A]

Sol. $H_2O \rightleftharpoons H^+ + OH^-$

$2H^+ + 2e^- \longrightarrow H_2(g)\uparrow$

(At cathode)

$2OH^- \rightarrow H_2O + H^+$

PART-B-MATHEMATICS

31. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Ans. **[A]**

Sol. $\vec{c} \cdot \vec{d} = 0$

$(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$5|\vec{a}|^2 + 6\vec{a}\vec{b} - 8|\vec{b}|^2 = 0$

$5 + 6 \cdot 1 \cdot 1 \cos\theta - 8 = 0$

$\cos\theta = \frac{3}{6} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

32. If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a \ln|\sin x - 2\cos x| + k$ then a is equal to :

(A) 1 (B) 2 (C) –1 (D) – 2

Ans. **[B]**

Sol. Differentiating both sides,

$\frac{5\tan x}{\tan x - 2} = 1 + \frac{a}{\sin x - 2\cos x}(\cos x + 2\sin x)$

$\frac{5\sin x}{\sin x - 2\cos x} - \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x} = 1$

$5\sin x - a(\cos x + 2\sin x) = \sin x - 2\cos x$

$4\sin x + 2\cos x = a(\cos x + 2\sin x)$

$a = \frac{2(2\sin x + \cos x)}{\cos x + 2\sin x} \Rightarrow a = 2.$

33. Consider the function, $f(x) = |x - 2| + |x - 5|$, $x \in R$.

Statement 1 : f '(D) = 0

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and f(B) = f(5).

(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 2.

(B) Statement 1 is true, Statement 2 is false.

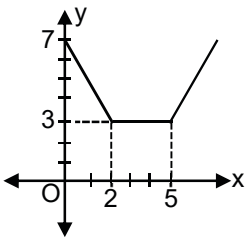
(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Ans. **[A]**

Sol. $f(x) = |x - 2| + |x - 5|$, $x \in R$

$$f(x)=\begin{cases}-2x+7, & x<2\\ 3, & 2\le x\le 5\\ 2x-7, & x>5\end{cases}$$



It is clear that f(x) is continuous in R and

differentiable in $(-\infty, 2)\cup(2, 5)\cup(5, \infty)$

$\therefore$ Statement 2 is correct.

Statement 1 is also correct but Statement 2 is not the correct explanation of Statement 1.

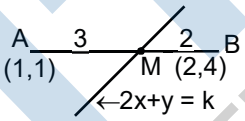
34. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(A) 6 (B) $\frac{11}{5}$ (C) $\frac{29}{5}$ (D) 5

Ans. **[A]**

Sol. Since, M divides A & B in the ratio 3 : 2.

$\therefore$ Coordinates of M are $\left(\frac{6+2}{5}, \frac{12+2}{5}\right)\equiv\left(\frac{8}{5}, \frac{14}{5}\right)$



M lies on the line 2x + y = k

$\therefore\ k=2\cdot\frac{8}{5}+\frac{14}{5}=6$.

35. **Statement 1 :** An equation of a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement 2 : If the line $y=mx+\frac{4\sqrt{3}}{m}$, $(m\neq 0)$ is a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 2.

(B) Statement 1 is true, Statement 2 is false.

(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Ans. **[D]**

Sol. Equation of tangent to parabola $y^2=16\sqrt{3}x$ is $y=mx+\frac{4\sqrt{3}}{m}$. A line y = mx + C is tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. When $c^2 = a^2m^2 + b^2$.

$$\frac{16\times 3}{m^2}=2m^2+4$$

$m^4 + 2m^2 = 24$

$\therefore$ On solving, $m^2 = 4$ or

$m^2 = -6$ (not possible)]

$\therefore \ m = \pm 2$

T : $y = 2x + 2\sqrt{3}$ & $y = -2x - 2\sqrt{3}$.

36. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is :

(A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{3}{8}$ (D) $\frac{1}{5}$

Ans. **[D]**

Sol. S : {1, 2, 3, 4, 5, 6, 7, 8}

$$P(E) = \frac{{}^4C_3}{{}^6C_3} = \frac{1}{5}.$$

37. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

(A) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (B) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (C) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (D) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

Ans. **[D]**

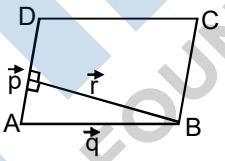
Sol. $\vec{r} = \lambda\vec{p} - \vec{q}$

$\vec{r}\cdot\vec{p} = (\lambda\vec{p} - \vec{q})\cdot\vec{p}$

$0 = \lambda\,\vec{p}\cdot\vec{p} - \vec{p}\cdot\vec{q}$

$\lambda = \frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}$

$\therefore \ \vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$.



38. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

(A) $x - 2y + 2z - 1 = 0$ (B) $x - 2y + 2z + 5 = 0$

(C) $x - 2y + 2z - 3 = 0$ (D) $x - 2y + 2z + 1 = 0$

Ans. **[C]**

Sol. Equation of plane parallel to $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z = \lambda$.

Distance from origin is 1.

$$\frac{|0+0+0-\lambda|}{\sqrt{1^2+2^2+2^2}} = 1$$

$\therefore \ \lambda = \pm 3$

P : $x - 2y + 2z = \pm 3$.

39. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{6}$

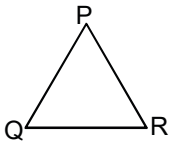
Ans. **[D]**

Sol. $3\sin P + 4\cos Q = 6$ .....(1)

$4\sin Q + 3\cos P = 1$ .....(2)

Square and add (1) & (2)

$24 \sin (P + Q) = 12$



$\therefore \quad \sin(P+Q) = \frac{1}{2}$

$\therefore \quad P+Q = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$

But when $P+Q = \frac{\pi}{6}$ then (1) & (2) not satisfied

$\therefore \quad P+Q = \frac{5\pi}{6} \quad \Rightarrow \quad R = \frac{\pi}{6}.$

40. If $f : R \to R$ is a function defined by $f(x) - [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

(A) discontinuous only at non-zero integral values of x.

(B) continuous only at x = 0.

(C) continuous for every real x.

(D) discontinuous only at x = 0.

Ans. **[C]**

Sol. $f : R \to R$

$f(x) - [x]\cos\left(\frac{2x-1}{2}\right)\pi$

$[\,] \to$ greatest integer function

When $x \in I$, then $f(x) = 0$

$\left[\because \quad \cos\left(\frac{2x-1}{2}\right)\pi = 0 \text{ for } n \in I\right]$

For $x \notin I$ then f(x) is product of two continuous function therefore it is continuous.

$\therefore$ f(x) is continuous for every real x.

41. **Statement 1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 +12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement 2 : $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 2.

(B) Statement 1 is true, Statement 2 is false.

(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Ans. **[D]**

Sol. **S-1:** 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 +12 + 16) + ... + (361 + 380 + 400)

Clearly number of terms in sequence are 20.

S-2 : $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$ is true.

$\therefore$ The sum of given series for 20 terms is $(20)^3 = 8000$.

42. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(A) $\frac{6}{5}$ (B) $\frac{5}{3}$ (C) $\frac{10}{3}$ (D) $\frac{3}{5}$

Ans. **[C]**

Sol. $(x-h)^2+(y-k)^2=k^2$

Centre (h, k), Radius = k

$(1-h)^2+k^2=k^2 \;\}\rightarrow h=1\;;\; k=\frac{5}{3}$

Radius $=\frac{5}{3}$. Diameter $=\frac{10}{3}$.

43. Let $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$, then $u_1 + u_2$ is equal to :

(A) $\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ (B) $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ (C) $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ (D) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$

Ans. **[B]**

Sol. $u_1=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$$\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$u_1=\begin{bmatrix}1\\-2\\1\end{bmatrix};\; u_2=\begin{bmatrix}1\\0\\0\end{bmatrix}$

$u_1+u_2=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$.

44. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is :

(A) an even positive integer

(B) a rational number other than positive integers

(C) an irrational number

(D) an odd positive integer

Ans. **[C]**

Sol. If n = 1, $\left(\sqrt{3}+1\right)^{2}-\left(\sqrt{3}-1\right)^{2}=4\sqrt{3}$.

An irrational number.

45. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :

(A) 630 (B) 879 (C) 880 (D) 629

Ans. **[B]**

Sol. (11) (10) (8) – 1 = 879.

46. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(A) $4x^2+y^2=8$ (B) $x^2+4y^2=16$ (C) $4x^2+y^2=4$ (D) $x^2+4y^2=8$

Ans. **[B]**

Sol. a = 4, b = 2

$\therefore$ Ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$

$x^2+4y^2=16$.

47. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to :

(A) $\frac{9}{2}$ (B) 0 (C) – 1 (D) $\frac{2}{9}$

Ans. **[A]**

Sol. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then $\begin{vmatrix}3-1 & k+1 & 0-1\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}2 & k+1 & -1\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$

$\Rightarrow$ 2 (3 – 8) – (k + 1) (2 – 4) – 1 (4 – 3) = 0

$\Rightarrow$ – 10 + 2k + 2 – 1 = 0

$\Rightarrow \quad 2k = 9 \Rightarrow k = \frac{9}{2}$.

48. Let a, b $\in$ R be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(A) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 2.

(B) Statement 1 is true, Statement 2 is false.

(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Ans. **[D]**

Sol. $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$

$f'(x) = \frac{1}{x} + 2bx + a$

extreme values at $x = -1, 2$

$\Rightarrow -1 - 2b + a = 0 \Rightarrow a - 2b = 1$ ...(1)

and $\frac{1}{2} + 4b + a = 0 \Rightarrow a + 4b = \frac{-1}{2}$ ...(2)

From (A) and (B) $a = \frac{1}{2}$, $b = \frac{-1}{4}$

again $f''(x) = 2b - \frac{-1}{x^2} = \frac{-1}{2} - \frac{1}{x^2}$

$\Rightarrow f''(-1) < 0$ and $f''(B) < 0$

$\Rightarrow$ f has local maximum at $x = -1$ and $x = 2$.

49. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

(A) either on the real axis or on a circle not passing through the origin.

(B) on the imaginary axis.

(C) either on the real axis or on a circle passing through the origin.

(D) on a circle with centre at the origin.

Ans. **[C]**

Sol. $\frac{z^2}{z-1}$ is real

Let $z = x + iy$

$\Rightarrow \quad \frac{z^2}{z-1} = \frac{x^2 - y^2 + 2ixy}{(x-1)+iy} = \frac{(x^2 - y^2 + 2ixy)(x-1-iy)}{(x-1)^2 + y^2}$

Now, imaginary part of $\frac{z^2}{z-1}$ equal to zero.

$\Rightarrow -y(x^2 - y^2) + (x-1)2xy = 0$

$\Rightarrow y(x^2 + y^2 - 2x) = 0$

$\Rightarrow y = 0$ or $x^2 + y^2 - 2x = 0$.

50. The negation of the statement

"If become a teacher, then I will open a school", is :

(A) Neither I will become a teacher nor I will open a school.

(B) I will not become a teacher or I will open a school.

(C) I will become a teacher and I will not open a school.

(D) Either I will not become a teacher or I will not open a school.

Ans. **[C]**

Sol. P = I be one a teacher.

Q = I will open a school.

$\sim (p \rightarrow q) = P \wedge \sim q$

$\Rightarrow$ I become a teacher and I will not open a school.

51. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals :

(A) $g(x) - g(\pi)$ (B) $g(x) \cdot g(\pi)$ (C) $\frac{g(x)}{g(\pi)}$ (D) $g(x) + g(\pi)$

Ans. **[A], [D]**

Sol. Given $g(x) = \int_0^x \cos 4t\, dt$

Now, $g(x+\pi) = \int_0^{x+\pi} \cos 4t\, dt = \int_0^{\pi} \cos 4t\, dt + \int_{\pi}^{x+\pi} \cos 4t\, dt$

$= \int_0^{\pi} \cos 4t\, dt + \int_0^{x} \cos 4t\, dt = g(\pi) + g(x)$

$\Rightarrow g(x + \pi) = g(x) + g(\pi)$

but $g(\pi) = 0$

$\therefore\ g(x + \pi) = g(x) + g(\pi) = g(x) - g(\pi)$

52. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) $\frac{9}{7}$ (D) $\frac{7}{9}$

Ans. **[A]**

Sol. Given $\frac{-dV}{dt} = 72\pi$

$t = 0$

Volume of gas = 4500π

$t = 49$ minute,

Volume of gas = $72\pi \times 49 = 3528\pi$

$\therefore$ After 49 minute volume of gas inside balloon = $(4500\pi - 3528\pi) = 972\pi$

$\therefore \quad \frac{4}{3}\pi r^3 = 972\pi \qquad \therefore \quad r = 9m$

Now, $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} - 72\pi = 4\pi(9^2)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{-2}{9}.$

53. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(A) exactly one real root. (B) exactly four real root.

(C) infinite number of real roots. (D) no real roots.

Ans. **[D]**

Sol. Let $e^{\sin x} = k$

$\therefore \quad k - \frac{1}{k} - 4 = 0 \;\; = 0 \Rightarrow k^2 - 4k - 1 = 0$

$k = e^{\sin x} = 2 + \sqrt{5}$ = It is greater than e

$\Rightarrow$ Not possible and $e^{\sin x} = 2 - \sqrt{5}$ = negative i.e. not possible.

$\therefore$ No solution.

54. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$, and $Y \cap Z$ is empty, is :

(A) 2^5 (B) 5^3 (C) 5^2 (D) 3^5

Ans. **[D]**

Sol. 1 can be distributed in two set Y and Z by 3 ways.

2 can be distributed in two set Y and Z by 3 ways.

3 can be distributed in two set Y and Z by 3 ways.

4 can be distributed in two set Y and Z by 3 ways.

5 can be distributed in two set Y and Z by 3 ways.

(The three ways are either only in Y or only in Z or in none of Y and Z.)

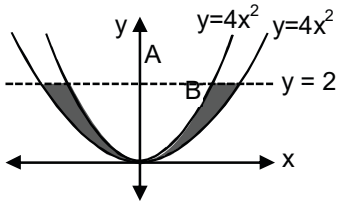
$\therefore$ Number of way of distributing each element = 3^5.

55. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line y = 2 is :

(A) $\frac{20\sqrt{2}}{3}$ (B) $10\sqrt{2}$ (C) $20\sqrt{2}$ (D) $\frac{10\sqrt{2}}{3}$

Ans. **[A]**

Sol. $\therefore$ Area $= 2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right) dy = \frac{20\sqrt{2}}{3}$.



56. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

(A) 0 (B) – 1 (C) – 2 (D) 1

Ans. **[A]**

Sol. $P^3 = Q^3$

$P^2Q = Q^2P$

$P^3 - P^2Q = Q^3 - Q^2P$

$P^2 (P - Q) = Q^2 (Q - P)$

$(P^2 + Q^2) (P - Q) = 0$

$| P^2 + Q^2 | | P - Q | = 0$

$| P^2 + Q^2 | = 0$ or $| P - Q | = 0$.

57. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement 1: Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement 2: Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.

(A) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1.

(B) Statement 1 is true, Statement 2 is false.

(C) Statement 1 is false, Statement 2 is true.

(D) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Ans. **[B]**

Sol. **S-2 :** Arithmetic Mean $= \frac{2x_1 + 2x_2 + ... + 2x_n}{n} = 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right) = 2\bar{x}$

$\therefore$ Statement-2 is false

Statement-1

We know variance of $x_1, x_2, x_3, \ldots\ldots x_n$.

$$\sigma = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

variance of $2x_1 ; 2x_2 ; \ldots\ldots\ldots ; 2x_n$

$$\sigma^2 = \frac{4\sum x_i^2}{n} - 4\left(\frac{\sum x_i}{n}\right)$$

$$= 4\left(\frac{\sum x_i^2}{n} - \frac{\sum x_i}{n}\right) = 4\sigma^2.$$

58. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\ p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is :

(A) $\frac{1}{2}\ln 18$ (B) ln 18 (C) 2 ln 18 (D) ln 9

Ans. **[C]**

Sol. $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$

$\therefore$ If $e^{\int \frac{-1}{2} dt} = e^{-t/2}$

$\therefore$ $p(t)\ e^{-t/2} = -450 \int e^{-t/2} dt + k$

$\therefore$ $p(t) = 900 + ke^{t/2}$ .....(A)

When t = 0 ; P(0) = 850

$\therefore$ k = – 50

$\therefore$ Equation (A) becomes

$p(t) = 900 - 50e^{t/2}$

$\therefore$ when p(t) = 0 then t = 2ln 18.

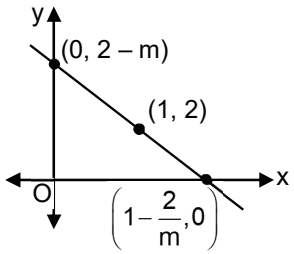
59. A line is drawn through the point (1, 2) to meet the coordinates axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ, is :

(A) – 2 (B) $\frac{-1}{2}$ (C) $\frac{-1}{4}$ (D) – 4

Ans. **[A]**

Sol. $\therefore$ Area $= \left|\frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m)\right|$

$= \frac{1}{2}\left\{m - 4 + \frac{4}{m}\right\}$



$\frac{dA}{dx} = 0$

Gives $m^2 = 4 \Rightarrow m = \pm 2$

$\therefore$ Area m = – 2.

60. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is :

(A) 150 (B) zero (C) – 150 (D) 150 times its 50^{th} term

Ans. **[B]**

Sol. 100 (a + 99d) = 50 (a + 49d)

$\therefore$ a + 149d = 0 .....(A)

and

T_{150} = a + 149d = 0 [From (A)].

PART-C-PHYSICS

61. This question has Statement-1 and statement-2. Of the four choices given after the Statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement -2 : $k_1 < k_2$

(A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

(B) Statement-1 is true, Statement-2 is true and Statement-2 is **not** the correct explanation of Statement-1

(C) Statement-1 is false, Statement-2 is true.

(D) Statement-1 is true, Statement-2 is false

Ans. **[A]**

Sol. Stretched by same force hence $k_1x_1 = k_2x_2$

More work is done on spring-1 hence

$$\frac{1}{2}k_1x_1^2 > \frac{1}{2}k_2x_2^2$$

$$\Rightarrow x_1 > x_2$$

$$\Rightarrow k_1 < k_2$$

62. This question has Statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinity is zero.

Statement-1 : When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\epsilon_0}$.

Statement -2 : The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3\epsilon_0}$.

(A) Statement-1 is false, Statement-2 is true.

(B) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

(C) Statement-1 is true, Statement-2 is true and Statement-2 is **not** the correct explanation of Statement-1

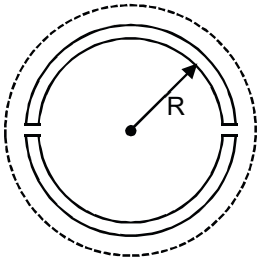
(D) Statement-1 is true, Statement-2 is false

Ans. **[A]**

Sol. $V_{anter} = \frac{3kQ}{2R}$, $V_{surface} = \frac{kQ}{R}$

$$\Delta U = q\Delta V = \frac{qkQ}{2R} = \frac{\rho R^2 q}{6\epsilon_0}$$

63. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is :

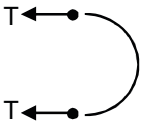


(A) $\pi SY\alpha\Delta T$ (B) $2SY\alpha\Delta T$ (C) $2\pi SY\alpha\Delta T$ (D) $SY\alpha\Delta T$

Ans. **[B]**

Sol. Thermal stress = $Y\alpha\Delta T$

Force developed = $YS\alpha\Delta T$



force exerted by the part = $2YS\alpha\Delta T$

64. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer)

(A) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ (B) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (C) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$ (D) $\frac{n^2h^2}{2(m_1+m_2)r^2}$

Ans. **[B]**

Sol. $I\omega = n\hbar$

Rotational energy $= \frac{1}{2}I\omega^2$ where $I = \mu r^2$

Putting we get $E = \frac{n^2\hbar(m_1+m_2)}{2m_1m_2r^2}$

65. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be :

(A) 5 (B) 6 (C) 2 (D) 3

Ans. **[B]**

Sol. No. of spectral lines = ${}^nC_2 = {}^4C_2 = 6$

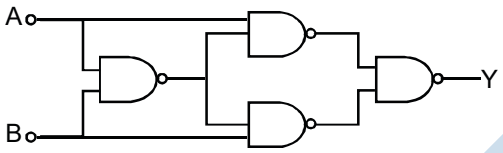
66. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance up to which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is :

(A) 40 km (B) 64 km (C) 80 km (D) 16 km

Ans. **[C]**

Sol. $d = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^3 \times 0.5} = 80\,km$

67. Truth table for system of four NAND gates as shown in figure is :



(A)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(B)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(C)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(D)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

Ans. **[C]**

68. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree

Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data:

(A) 58.65 degree (B) 59 degree (C) 58.59 degree (D) 58.77 degree

Ans. **[A]**

Sol. Least count $(LC) = \dfrac{0.5 \text{ degree}}{30}$

Reading = Main scale reading + vernier scale reading

$= 58.5 + 9 \times \dfrac{0.5}{30}$

= 58.65 degree

69. This question has Statement-1 and statement-2. Of the four choices given after the Statements, choose the one that best describes the two statements.

Statement-1 : Davisson - Germer experiment established the wave nature of electrons.

Statement-2 : If electrons have wave nature, they can interfere and show diffraction.

(A) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true and Statement-2 is **not** the correct explanation of Statement-1

(C) Statement-1 is false, Statement-2 is true.

(D) Statement-1 is true, Statement-2 is false

Ans. **[A]**

Sol. $F=K_1S_1=K_2S_2$

$W_1=FS_1$, $W_2=FS_2$

$K_1S_1^2 > K_2S_2$

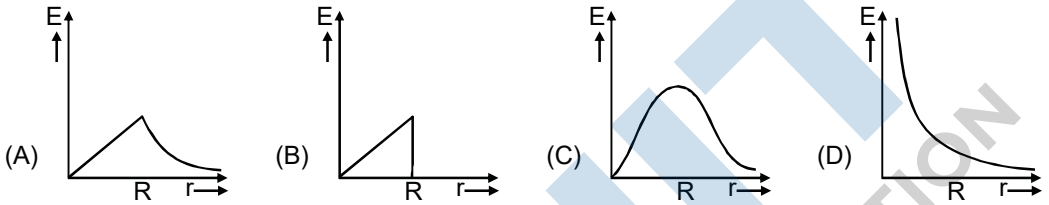
$S_1 > S_2$

$K_1 < K_2$

$W \propto K$

$W_1 < W_2$

70. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be :



Ans. **[A]**

Sol. $E_{inside} \propto r$

$E_{outsides} \propto \frac{1}{r^2}$

71. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now:

(A) $\frac{3f}{4}$ (B) 2f (C) f (D) $\frac{f}{2}$

Ans. **[C]**

Sol. $f = \frac{v}{\lambda};\ \lambda = 2L$

$f' = \frac{v}{\lambda};\ \frac{\lambda}{4} = \frac{L}{2} \Rightarrow \lambda = 2L$

hence $f' = f$

72. If a simple pendulum has Significant amplitude (up to a factor of 1/ e of original) only in the period between t = 0s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds:

(A) $\frac{1}{b}$ (B) $\frac{2}{b}$ (C) $\frac{0.693}{b}$ (D) b

Ans. **[A]**

Sol. $a = -bv$

hence $v = v_0 e^{-bt}$

comparing with $N = N_0 e^{-\lambda t}$

Average life time $= \frac{1}{\lambda} = \frac{1}{b}$

73. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :

(A) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(B) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

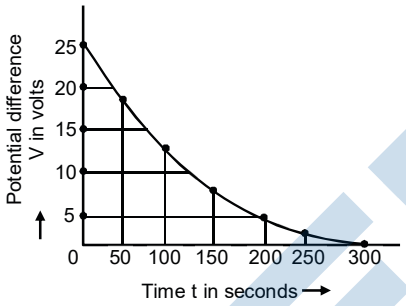
(C) development of air current when the plate is placed.

(D) induction of electrical charge on the plate

Ans. **[B]**

Sol. Oscillating coil produces time variable magnetic field. It cause eddy current in the aluminium plate which causes anti–torque on the coil, due to which is stops.

74.



The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between:

(A) 50 sec and 100 sec

(B) 100 sec and 150 sec

(C) 150 sec and 200 sec

(D) 0 and 50 sec

Ans. **[B]**

Sol. $V = Ve^{-t/\tau}$

at $t = 200$ sec, $V = 5$, $V_0 = 25$

hence we get $\tau = 124.2$ sec

75. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :

(A) 750 K

(B) 600 K

(C) efficiency of Carnot engine cannot be made larger than 50%

(D) 1200 K

Ans. **[A]**

Sol. $0.4 = 1 - \frac{T_{sink}}{500} \Rightarrow T_{sink} = 300\ K$

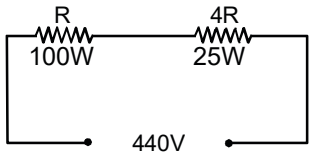
$0.6 = 1 - \frac{300}{T_{source}} \Rightarrow T_{source} = 750\ K$

76. Two electric bulbs marked 25W-220V and 100W-220V are connected in series to a 440V supply. Which of the bulbs will fuse?

(A) 25W (B) neither (C) both (D) 100 W

Ans. **[A]**

Sol.



77. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by and $\vec{X}$ that of wave propagation by $\vec{k}$. Then :

(A) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{E} \times \vec{B}$ (B) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{B} \times \vec{E}$

(C) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{B} \times \vec{E}$ (D) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{E} \times \vec{B}$

Ans. **[D]**

78. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :

(A) 6.4×10^9 Joules (B) 6.4×10^{10} Joules

(C) 6.4×10^{11} Joules (D) 6.4×10^8 Joules

Ans. **[B]**

Sol. On surface of earth $U = -\frac{GmMe}{Re}$ $mgRe = -6.4 \times 10^{10}$ Joule

79. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by :

(A) $\frac{I_m}{5}\left(1 + 4\cos^2\frac{\phi}{2}\right)$ (B) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

(C) $\frac{I_m}{9}(4 + 5\cos\phi)$ (D) $\frac{I_m}{3}\left(1 + 2\cos^2\frac{\phi}{2}\right)$

Ans. **[B]**

Sol. I_0 and $4I_0$

$I_m = 9I_0$

$I_{res} = I_0 + 4I_0 + 2\sqrt{4I_0^2}\cos\phi$

$= 5I_0 + 4I_0\cos\phi$

$= \frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

80. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be :

(A) $10\sqrt{2}$ m (B) 20 m (C) $20\sqrt{2}$ m (D) 10 m

Ans. **[B]**

Sol. $\frac{u^2}{2g} = 10m$

$R_{max} = \frac{u^2}{g} = 20m$

81. Assume that a neutron breaks into a proton and an electron. The energy released during this process is:

(Mass of neutron = 1.6725×10^{-27} kg

Mass of proton = 1.6725×10^{-27} kg

Mass of electron = 9×10^{-31} kg)

(A) 6.30 MeV (B) 5.4 MeV (C) 0.73 MeV (D) 7.10 MeV

Ans. **NO ANSWER (WRONG DATA)**

but correct answer is [C] with actual data.

82. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?

(A) 3.2 m (B) 5.6 m (C) 7.2 m (D) 2.4 m

Ans. **[B]**

Sol. $\frac{1}{12}+\frac{1}{240}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{7}{80}$

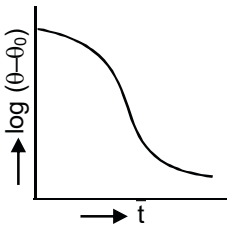
Δx due to slab $= t\left(1-\frac{1}{\mu}\right) = \frac{1}{3}CM$

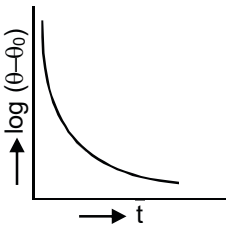
* New v should be $= 12-\frac{1}{3} = \frac{35}{3}CM$

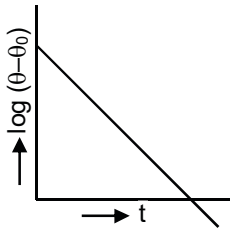
* $\frac{1}{u}=\frac{1}{v}\frac{1}{f}=\frac{3}{35}\frac{7}{80}=-\frac{1}{500}$

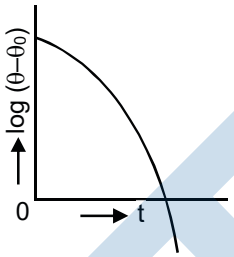
u = –5.6 m

83. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :

(A) 

(B) 

(C) 

(D) 

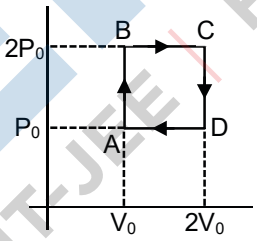
Ans. **[C]**

Sol. $\frac{dT}{dt} = -k(T - Ts)$

$$\ln\left|\frac{T - T_s}{T_0 - T_s}\right| = -kt$$

$\ln(T - T_s) = \ln(T_0 - T_s) - kt$

84. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly: (Assume the gas to be close to ideal gas)



(A) 10.5% (B) 12.5% (C) 15.4% (D) 9.1%

Ans. **[3]**

Sol. $Q_{AB} = nCv\left[\frac{p_0v_0}{R}\right]$, $Q_{BC} = nCp\left[\frac{2P_0v_0}{R}\right]$

$Q_{CD} = nCv\left[\frac{2P_0v_0}{R}\right]$, $Q_{DA} = nCp\left[\frac{p_0v_0}{R}\right]$

$Q_{repected} = Q_0 + Q_{DA}$

$Q_{absorbed} = Q_{AB} + Q_{BC}$

$\eta = 1 - \frac{Q_{rej}}{Q_{ab}} = 0.154$

85. Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct?

(A) $r_\alpha > r_d > r_p$ (B) $r_\alpha = r_d > r_p$ (C) $r_\alpha = r_p = r_d$ (D) $r_\alpha = r_p < r_d$

Ans. **[D]**

Sol. $R = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

$R_P : R_d : R_\alpha = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e}$

$= 1 : \sqrt{2} : 1$

86. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is :

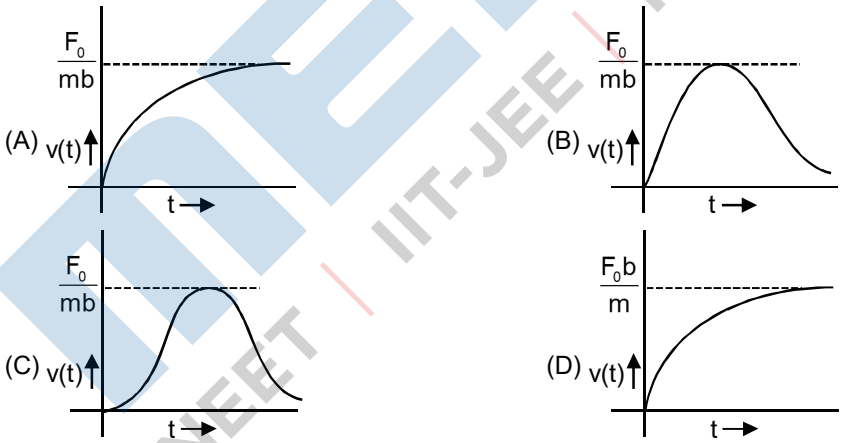
(A) 1% (B) 3% (C) 6% (D) zero

Ans. **[C]**

Sol. $R = \frac{V}{I}$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = 6\%$

87. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves?



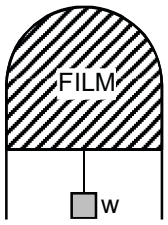
Ans. **[A]**

Sol. $a = \frac{f_0 e^{-bt}}{m}$

$\int_0^v dv = \int_0^t \frac{f_0}{m} e^{-bt}\, dt$

$v = \frac{f_0}{mb}(1 - e^{-bt})$

88. A thin liquid film formed between aU-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is:



(A) 0.05 Nm^{-1} (B) 0.025 Nm^{-1} (C) 0.0125 Nm^{-1} (D) 0.1 Nm^{-1}

Ans. **[B]**

Sol. 2SL = weight

S = 0.025 N/m

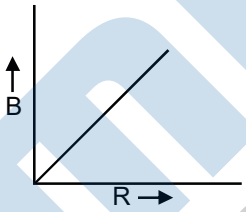
89. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :

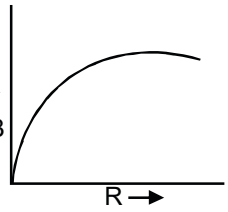
(A) $r_1 : r_2$ (B) 1 : 1 (C) $m_1r_1 : m_2 r_2$ (D) $m_1 : m_2$

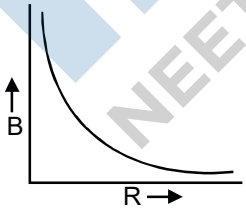
Ans. **[A]**

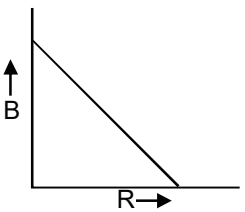
Sol. $\frac{a_1}{a_2} = \frac{r_1\omega^2}{r_1\omega^2} = \frac{r_1}{r_2}$

90. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure.

(A)


(B)


(C)


(D)


Ans. **[C]**

Sol. $dB = \frac{\mu_0 dI}{2r}$

Integrating we get $B \propto \frac{1}{R}$

$$dp = \sigma(2\pi r dr) = \frac{2Qrdr}{R^2}$$

$$dI = \frac{(dq)\omega}{2\pi} = \frac{Q\omega rdr}{\pi R^2}$$